In memory of Filippo Spagnolo

Towards cultural analysis of content: problems with variation in primary school

Mariolina Bartolini Bussi

Rita Canalini – Franca Ferri

Università di Modena e Reggio Emilia (Italia)

SEMT '11
International Symposium
Elementary Mathematics Teaching

August 21 - 26, 2011, Prague, Czech Republic
Charles University in Prague, the Faculty of Education

Laboratorio delle Macchine Matematiche

Università degli Studi di Modena e Reggio Emilia
Ateneo fondato nel 1175

Mathematics teacher education …

The mathematical knowledge needed for teaching in elementary schools

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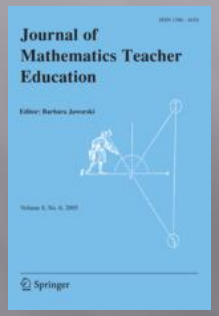

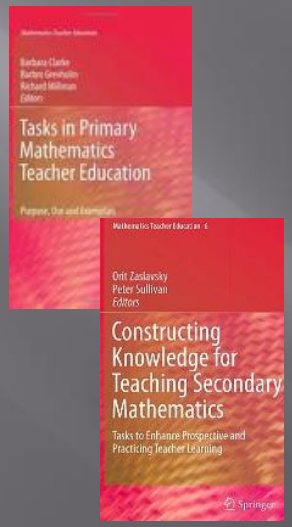

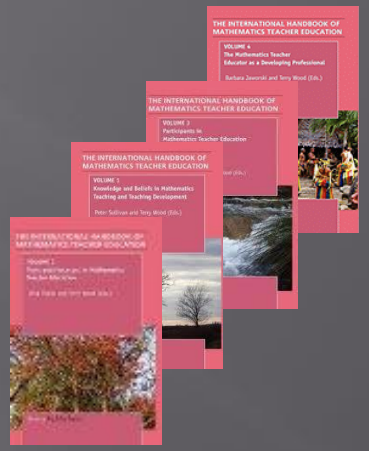

Research fora

Mathematics teacher education …

Anna Sfard
Hans Freudenthal medal
2007 ICMI

ICME 11 Mexico 2008
11th International Congress on Mathematical Education

I am pleased to find out that the last few years have been the era of the teacher as the almost uncontested focus of researchers' attention [...]. And we have certainly come a long way since the era of the curriculum, roughly corresponding to the 1960s and 1970s when the main players in the educational game were the developer and the textbook.
I consider the re-conceptualization of the relationship between the teacher and the researcher a big leap toward research that plays a genuine role in shaping and improving practice

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- Cultural Analysis of Content
- Chinese problems with variation
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  - ✓An Italian transposition (Franca Ferri)
- Multiplicative Problems
  - ✓An Italian transposition (Rita Canalini)
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- Open problems

Cultural Analysis of Content

means

Presenting mathematics as an evolving discipline, with different levels of rigor both at a specific moment in history (according to the cultural environment and specific needs), and across history, and as a domain of culture as a set of interrelated cultural tools and social practices, which can be inherited over generations.[...] It can lead teachers to radically question their beliefs concerning mathematics in general and specific subject matter in particular.

(Boero & Guala, 2008)

Word Problems in Primary School: *biànshì* problems

biànshì 变式 *variation*

(according to the Chinese "indigenous" meaning)

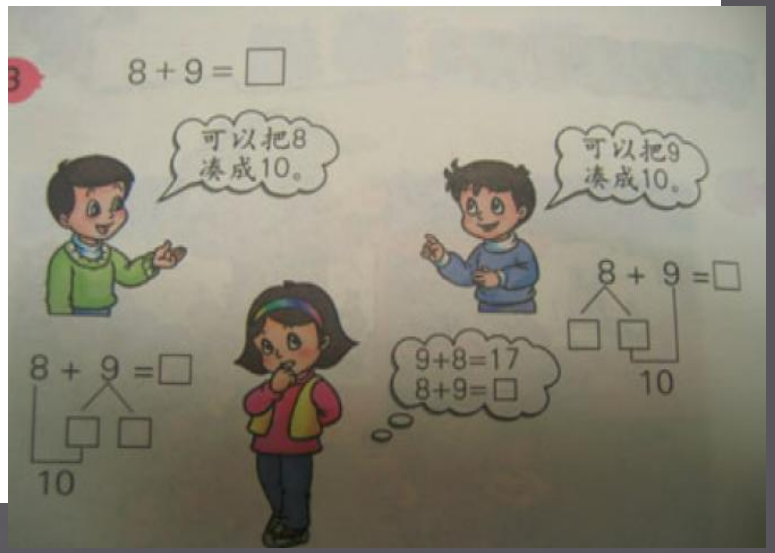
OPMS	One Problem Multiple Solutions *varying solutions*
OPMC	One Problem Multiple Changes *varying conditions and conclusions*
MPOS	Multiple Problem One Solution *varying presentations – exercises*

Xuhua Sun (2011)

Word Problems in Primary School: *biànshì* problems

An example from first grade (Xuhua Sun, 2011)

OPMS One Problem Multiple Solutions
varying solutions



Word Problems in Primary School: *biànshì* problems

MPOS Multiple Problem One Solution
varying presentations – exercises

Word Problems in Primary School: *biànshì* problems

biànshì 变式 *variation*

(according to the Chinese “indigenous” meaning)

OPMC One Problem Multiple Changes
varying conditions and conclusions

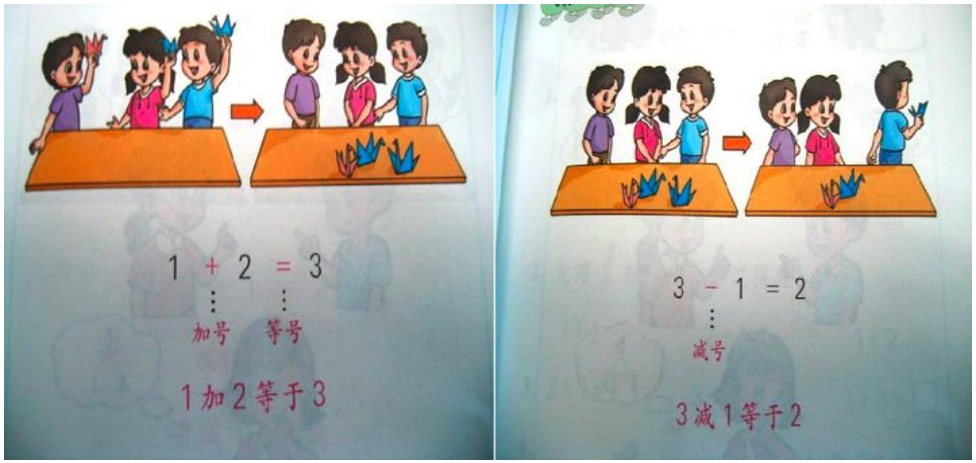
Concept / meaning construction

Addition-subtraction

Word Problems in Primary School: *biànshì* problems

An example from first grade (Xuhua Sun, 2011)

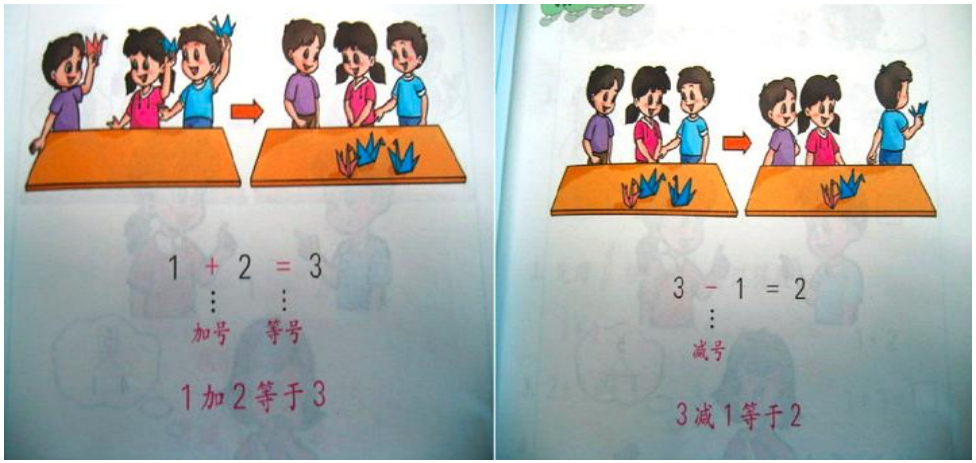
OPMC One Problem Multiple Changes
varying conditions and conclusions



Word Problems in Primary School: *biànshì* problems

Is this a prompt for “Western” primary school teachers?

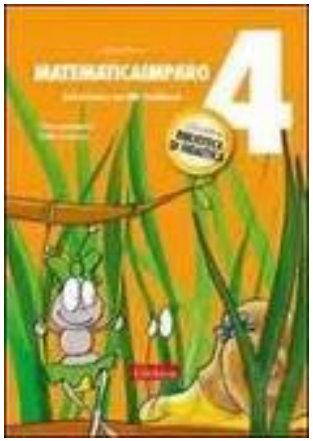
Is this a prompt for Italian primary school teachers?



Word Problems in Primary School: *biànshì* problems

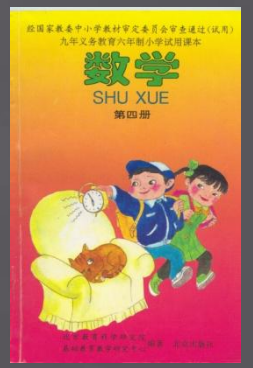
Is this a prompt for Italian primary school teachers?
Not only different chapters but even different booklets with different authors in the same popular series!

Addition



Subtraction

The prompt for additive word problems nine problems on ducks in a river



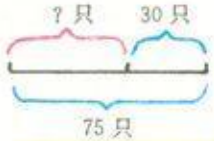
1996

例 11 先解答，再说一说横着、竖着每组三道题有什么联系？

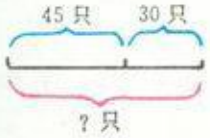
1. (1)河里有 45 只白鸭，30 只黑鸭，一共有多少只鸭？

(2)河里有白鸭和黑鸭一共 75 只，其中 45 只是白鸭，有多少只黑鸭？

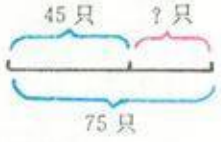
(3)河里有白鸭和黑鸭一共 75 只，有 30 只黑鸭，有多少只白鸭？



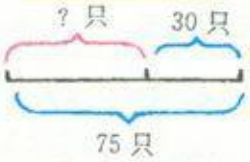
2. (1)河里有一群鸭，游走 30 只，还剩 45 只。这群鸭有多少只？



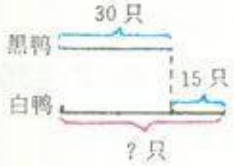
(2)河里有 75 只鸭，游走一些，还剩 45 只，游走多少只？



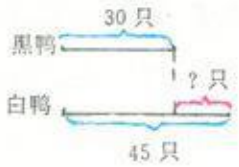
(3)河里有 75 只鸭，游走 30 只，还剩多少只？



3. (1)河里有 30 只黑鸭，白鸭比黑鸭多 15 只(黑鸭比白鸭少 15 只)，白鸭有多少只？



(2)河里有 30 只黑鸭，45 只白鸭，白鸭比黑鸭多几只？(黑鸭比白鸭少几只)



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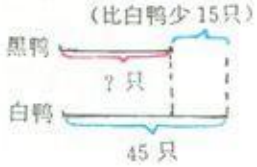
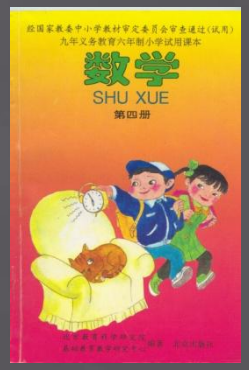


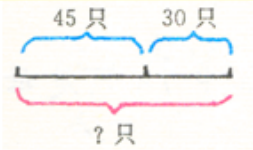


2° grade
at the end
of the year

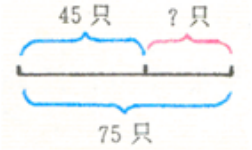
1996

First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships

(1)**In the river there are** 45 white ducks and 30 black ducks. All together how many ducks are there?



(2)**In the river there are** white ducks and black ducks. All together there are 75 ducks. 45 are white ducks. How many black ducks are there?



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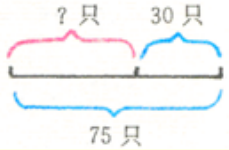
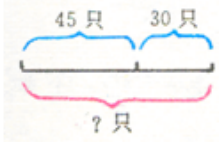
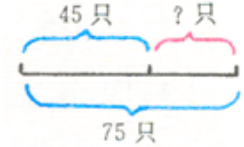


(1)**In the river there is** a group of ducks. 30 ducks swim away. 45 ducks are still there. How many ducks are in the group (at the beginning)?



(2)**In the river there are** 75 ducks. Some ducks swim away. There are still 45 ducks. How many ducks have swum away?



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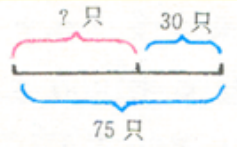

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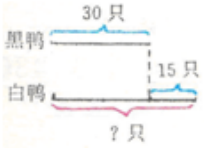

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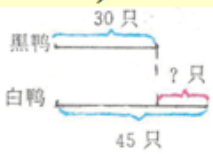

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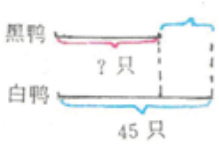

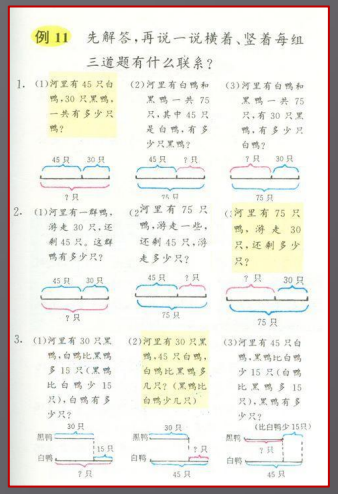

1996

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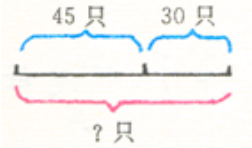	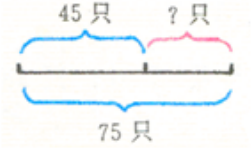	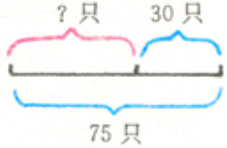
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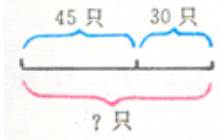	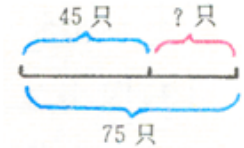	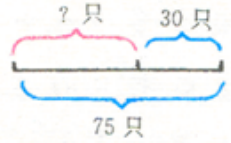
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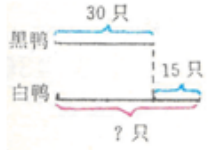	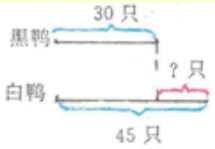	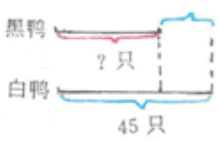

In each row

PROBLEM

VARIATION 1

VARIATION 2

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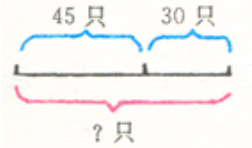	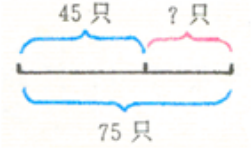	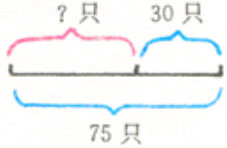
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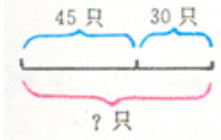	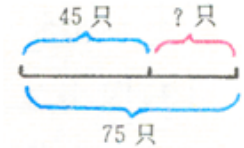	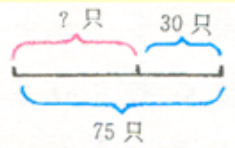
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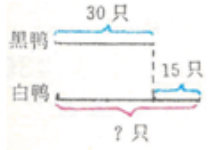	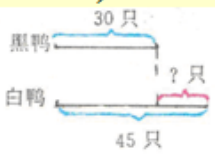	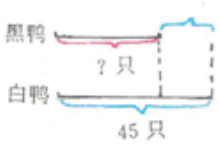

We know it!

Combine

Change

Compare

e. g. Carpenter et al.

Research and School traditions (in the "West")

Research:
A lot of "semantic" studies and classifications of problems (since the 70s)

Curricula,
Textbooks
and Schools
Little impact (if any)
Distractor effects
Low performances

Combine

Change

Compare

e. g. Carpenter et al.

In China
(Japan, Korea and Far East)

Variation (with shift from one problem to another)
seems to be
the standard way
to approach problem solving

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to approach problem solving

Chinese students are better problem solvers
(in crowded classrooms!) than others!

(see Shanghai and Hong Kong cases
in OECD PISA, 2009)

In China (Japan, Korea and Far East)

Variation (with shift from one problem to another)
seems to be
the standard way
to approach problem solving

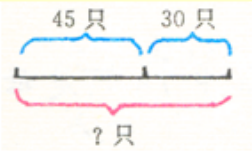
?

Chinese students are better problem solvers
(in crowded classrooms!) than others!

(see Shanghai and Hong Kong cases
in OECD PISA, 2009)

First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships

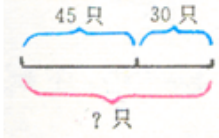
(1)**In the river there are** 45 white du[illegible]s and 30 black ducks. A[illegible]gether how many ducks there?



(2)**In the river there are** white ducks and black ducks. All together the[illegible] are 75 ducks. 45 are w[illegible] ducks. How many bla[illegible] are there?

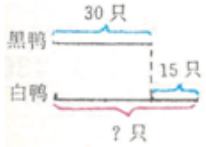
(3)**In the river there** white ducks and bla[illegible] ducks. All togeth[illegible]re 75 ducks. 30[illegible]cks. How m[illegible]s are th[illegible]

(1)**In the river there is** a group of ducks. 30 duck[illegible] swim awa[illegible] still there. H[illegible] ducks are in the [illegible] the beginning)?

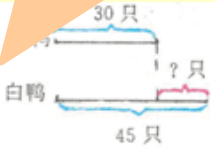


(2)**I**[illegible]

(1)**In the river ther**[illegible] 30 black ducks. [illegible] ducks are 15[illegible] black ducks (black ducks are 15 less than white ducks). How many white ducks are there?



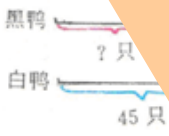
white duc[illegible] white du[illegible] black du[illegible] black d[illegible]an white d[illegible]



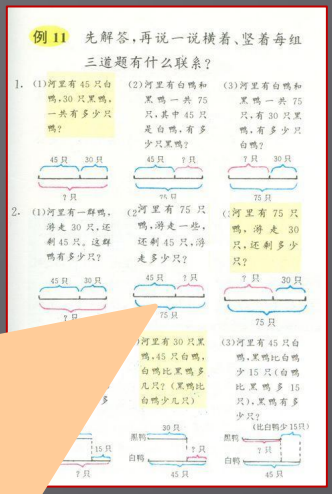
[illegible]n.

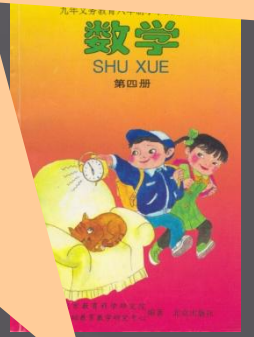
[illegible]s). H[illegible]k

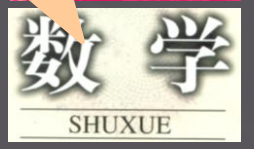
[illegible]s are th[illegible]



A prompt
and
a challenge
for Italian teachers
in second grade!







1996

Working with teachers (since 2007)

4 workshops with prospective teachers
(~ 25 participants x workshop)

2 workshops with practicing teachers
(~ 25 participants x workshop)

Collaboration
with 2 teacher-researchers:
Rita Canalini and Franca Ferri
(the two coauthors of this study, engaged also in the workshops as teacher educators)

First teaching experiment: nine problems on ducks in a river Italian transposition (2° grade) Franca Ferri

22 pupils

First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships

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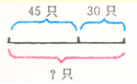	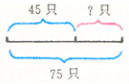	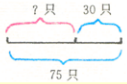
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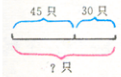	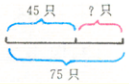	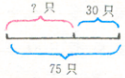
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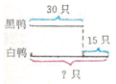
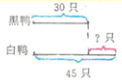
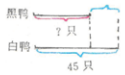
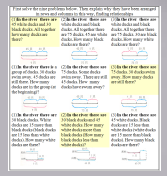
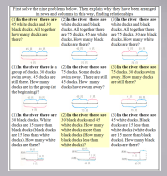
		



First teaching experiment (Franca Ferri)

Summary

1. Solution of the nine Chinese problems (without schemes)
2. Creation of three problems like the Chinese ones of the first row
3. Creation of three problems with the same arithmetic typology of the Chinese problems of the second row
4. Pairing the nine schemes with the nine Chinese problems



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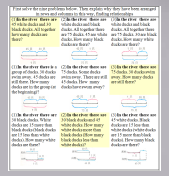

First task

Observe, reflect and solve (Chinese problems)

(1)**In the river there are** 45 white ducks and 30 black ducks. All together how many ducks are there?	(2)**In the river there are** white ducks and black ducks. All together there are 75 ducks. 45 are white ducks. How many black ducks are there?	(3)**In the river there are** white ducks and black ducks. All together there are 75 ducks. 30 are black ducks. How many white ducks are there?
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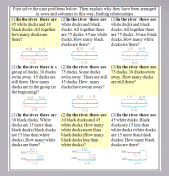


First task

Results

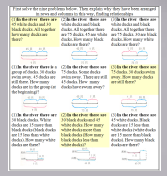
(individual work)

18 pupils are present.

9 pupils solve all the 9 problems.

5 pupils make some mistake.

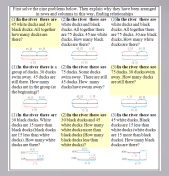
4 pupils do not solve the problems in the third row.



First task

Mathematical Discussion (starter)

Teacher*: You have solved well or badly the problems I have given. I have told you that there are Chinese, as they have been translated from a Chinese textbook for the second grade. I have given you nine problems and you might have thought that I went mad* ...[she smiles]
Lorenzo [with emphasis]*: We have never solved so many problems all together.*
Teacher*: I have given them all together, because they were together in the Chinese textbook and also because I have thought that they were a bit special and could stay together.*
Mohassen*: It's true!*
Teacher*: There, I'd like to understand what you have seen, raise your hands. Observe them carefully, reflect and come in.*

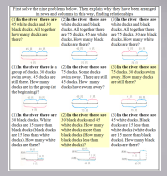


First task

Mathematical Discussion: outcomes

There are always the same numbers (at least in the first two rows).
There are always the same operations (additions, subtractions).
Subtractions are "additions with dots": "... + 45 = 75".
In the first column only additions; then only subtractions.
Every problem has the solution in another problem.

You had to understand that everything was linked. Also the last ones, that were different, were linked to the same story of ducks. The numbers were always the same (Arianna).



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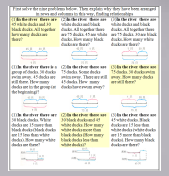

Second task

Invent three problems like the Chinese ones of the first row.

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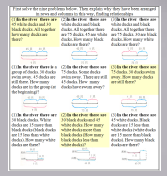

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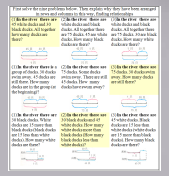
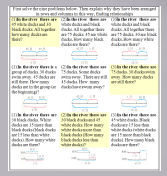


Third task

This is the second row of the Chinese problems. Try to construct three similar ones, maintaining the same arithmetical typology.

(1) **In the river there is** a group of ducks. 30 ducks swim away. 45 ducks are still there. How many ducks are in the group (at the beginning)?	(2) **In the river there are** 75 ducks. Some ducks swim away. There are still 45 ducks. How many ducks have swum away?	(3) **In the river there are** 75 ducks. 30 ducks swim away. How many ducks are still there?



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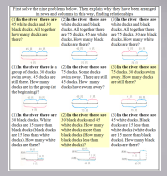

Fourth task

Observe, try to understand what the schemes mean, pair them with the nine problems (cut and paste the right scheme below each problem)
and explain why.

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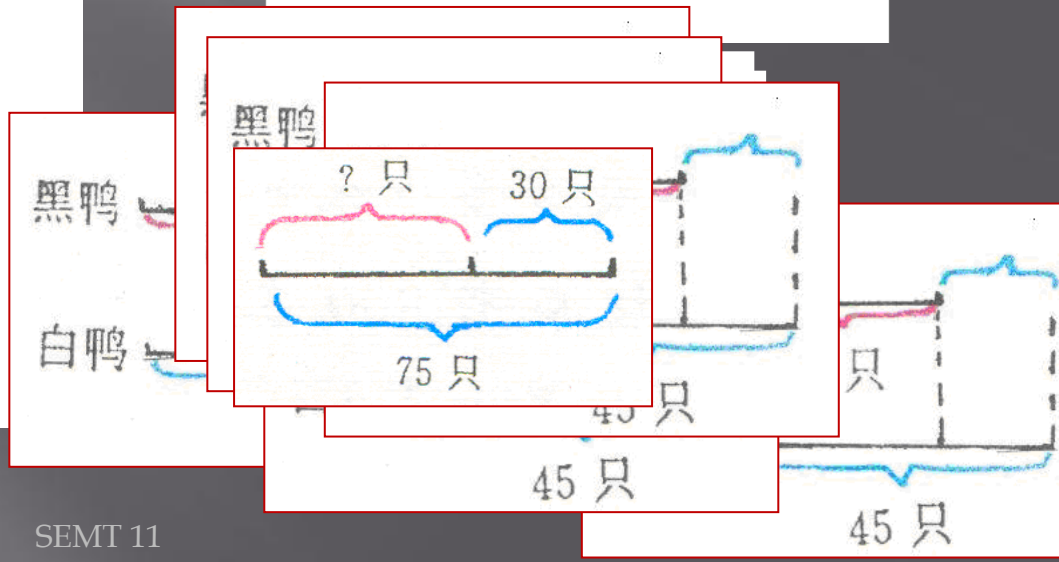
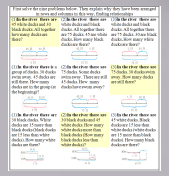
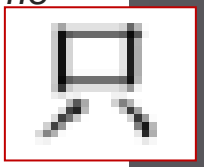




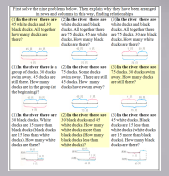
Fourth task

Mathematical Discussion (introduction of the task)

Teacher: *Many have said that choosing the scheme for the first problem is quite easy. Why? How have you understood which scheme matches the first problem?*
Sofia: *If you look carefully, you see that the scheme is equal to the text; in both we have 45 white ducks and 30 black ducks and we are asked how many all together and in the scheme there is a question mark.*
Lorenzo: *The Chinese word* [the classifier or measure word zhī] *close to the numbers might mean ducks, we don't know, it is always the same and the text is about ducks.*



Fabio: *In the scheme the parentheses are longer when the number is greater and shorter if the number is smaller. They are not equal: there are long ones, medium ones and short ones.*
Mohassen: *The red parenthesis in the first text means that the result is larger than the two numbers, as it is longer.*
Lorenzo: *If you look, you see that the red parenthesis is the number to be found, as there is always a question mark.*
Arianna: *The two blue parentheses represent the data, whilst the red one is the number to be found. There is a scheme for each problem. Yet there are equal schemes, because there problems that are solved in the same way.*
Teacher: *With this indications/observations try to pair each text with its scheme. Good work!*

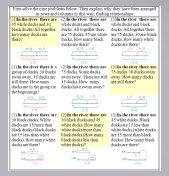


Fourth task

Results.

All the pupils solve the task correctly although in different times.

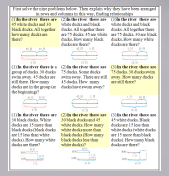
In a limited number of cases the teacher helps for the problems of the third row.



Fourth task

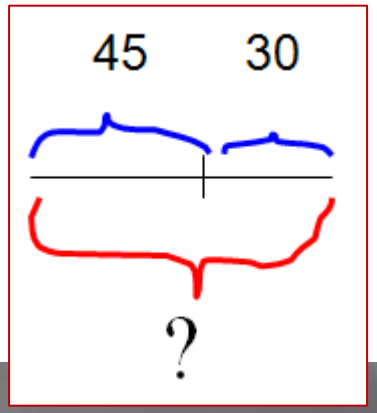
It was easy to find the schemes: if red was small, it is the smallest number, i. e. 30, or even 45 that is a bit larger. If red is even larger, it is the largest, i. e. 75.
Hence I believe that it was clear by reading and looking at the red one.
I have helped myself also looking at the blue, to find the other numbers.

Colours and sizes

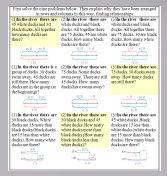


Fourth task

When I am doubtful, I look at the scheme and write the operation: the blue lines are the numbers written in the text, the red line is the result. Look at this example:



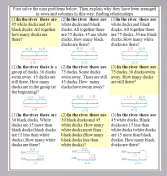
Colours and operations



Fourth task

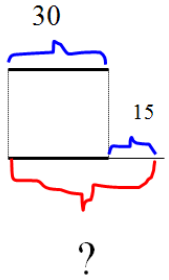
The first three were very easy, the next three were easy, the last three were a bit difficult.
To find the schemes I followed the previous operations. For instance, I had repeated twice the operation 45 + 30 or 30 + 45. I took the same scheme.
This for the first six problems.

Schemes and operations



Fourth task

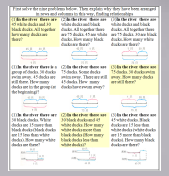
For the 7 problem I have thought: the operation is 30+15=45. Let us choose this scheme because there is 30 that joins 15 with the red line. It is OK.



For the 8 problem there is 30 that takes a piece of 45; the rest is taken by the red line. Hence 45 – 30 = 15

Schemes and operations

A part is taken by …



Fourth task

Results.

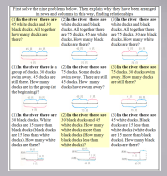
All the pupils solve the task correctly although in different times.

In a limited number of cases the teacher helps for the problems of the third row.

The strategies used by the pupils are mostly based on the length and the colour of parentheses.

Most pupils become aware that if a+b=c then c-a=b and c-b=a, independently from the specific value of the numbers a,b,c.

They express algebraic reasoning (rather than arithmetic reasoning).



First teaching experiment (Franca Ferri)

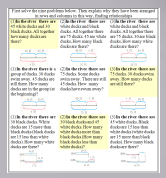
Summary

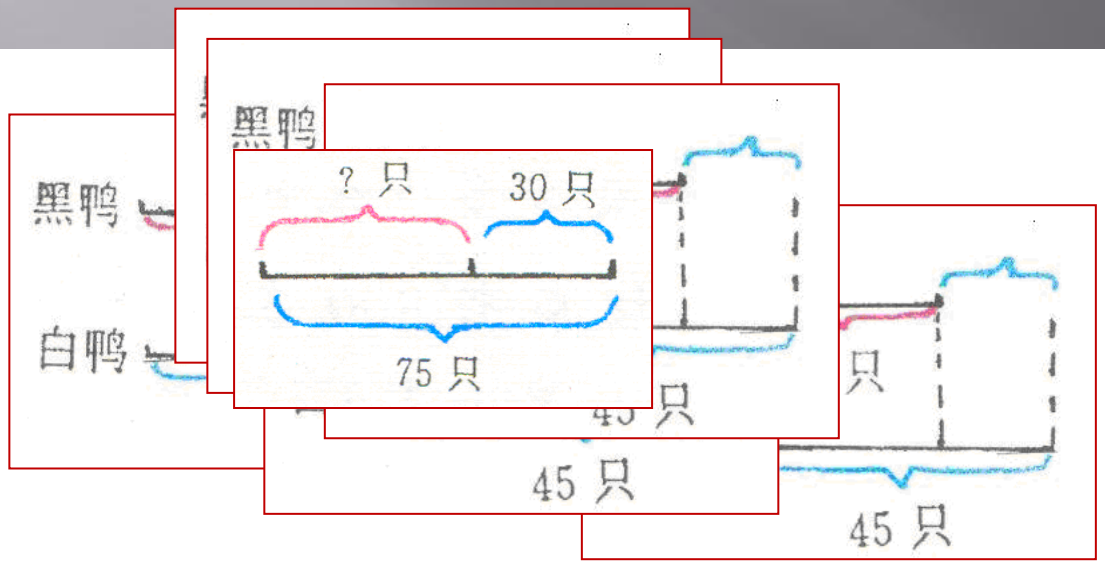
1. Solution of the nine Chinese problems (without schemes)
2. Creation of three problems like the Chinese ones of the first row
3. Creation of three problems with the same arithmetic typology of the Chinese problems of the second row
4. Pairing the nine schemes with the nine Chinese problems

SEMANTIC

SYNTAX

First teaching experiment (Franca Ferri)

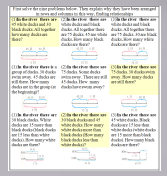




The schemes are SEMIOTIC MEDIATORS for word problems (and the solving strategies)

4. Pairing the nine schemes with the nine Chinese problems

SYNTAX



First teaching experiment (Franca Ferri)

What followed?

3. Experiments on multiplicative problems

Word Problems in Primary School: *biànshì* problems

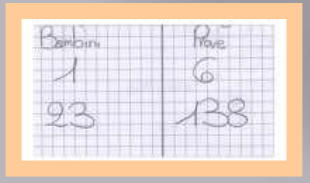
biànshì 变式 *variation*

(according to the Chinese “indigenous” meaning)

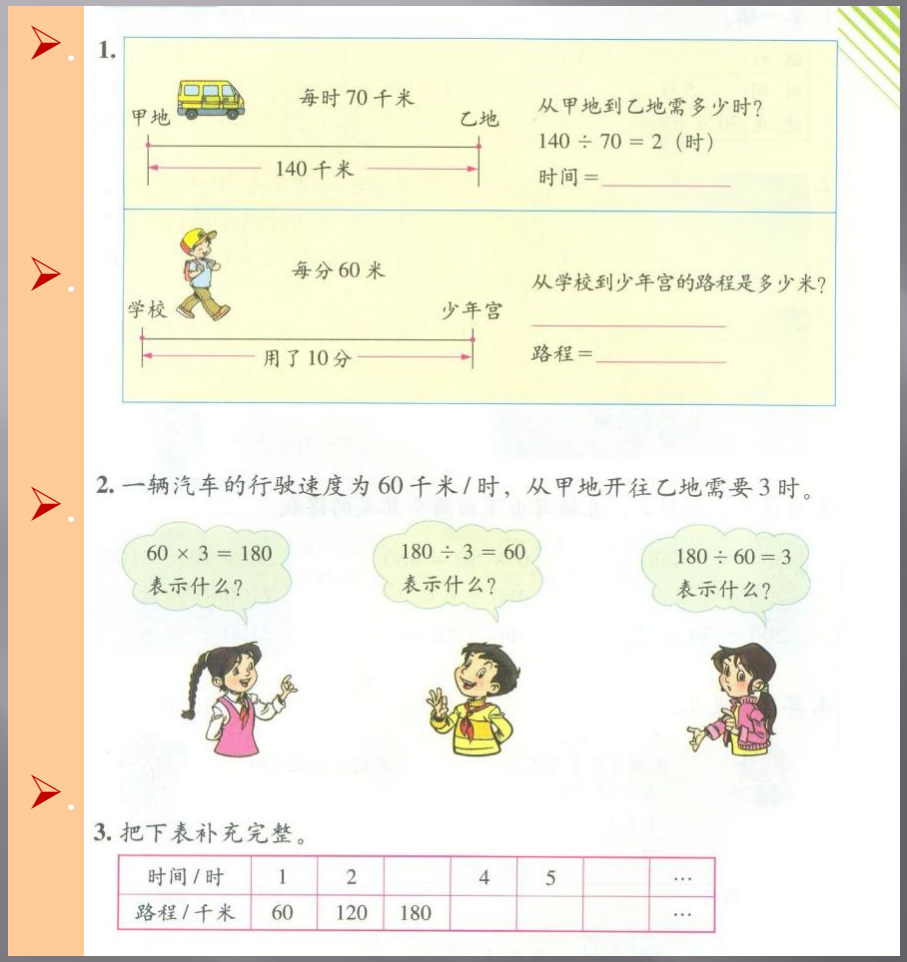
OPMC One Problem Multiple Changes
varying conditions and conclusions

Concept / meaning construction

Multiplication-division



Second teaching experiment: multiplicative problems (4° grade) Rita Canalini



1.

甲地 — 每时 70 千米 — 乙地

140 千米

从甲地到乙地需多少时？

140 ÷ 70 = 2（时）

时间 = _____

学校 — 每分 60 米 — 少年宫

用了 10 分

从学校到少年宫的路程是多少米？

_____

路程 = _____

2. 一辆汽车的行驶速度为 60 千米 / 时，从甲地开往乙地需要 3 时。

60 × 3 = 180 表示什么？

180 ÷ 3 = 60 表示什么？

180 ÷ 60 = 3 表示什么？

3. 把下表补充完整。

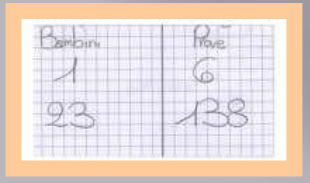
时间 / 时	1	2		4	5		…
路程 / 千米	60	120	180				…

A prompt from a Chinese textbook (4 grade)

- A bus covers 70 km in 1 hour. How many hours are needed to go from A to B (140 kms)?
- A child covers 60 m in 1 minute. It takes 10 minutes to go from school to aftershool. How far is afterschool from school?
- A car travels with a speed of 60 km per hour. It takes 3 hours to go from A to B. What does each operation mean?
- Fill the following table

time/hour

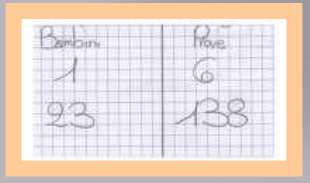
distance/km



Second teaching experiment: multiplicative problems (4° grade)

Rita Canalini

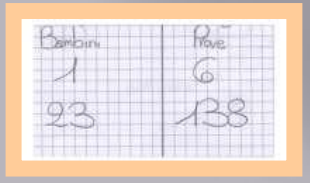
23 pupils



Second teaching experiment (Rita Canalini)

Summary (of the initial part)

0. Experience on additive problems and multiplicative problems: fostering the production of solving schemes. (3 grade)
1. Teacher Anna and admission tests: Interpretation, completion and solution of a system of three problems (Chinese style).
2. Mario and the photo album: Exploiting Vergnaud's scheme.



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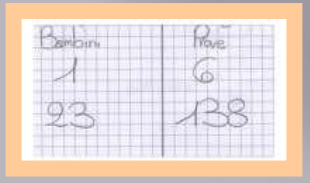


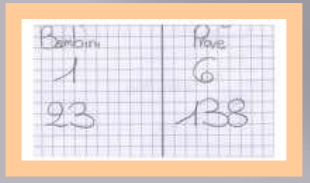
First task

1) Read very carefully the texts and complete them with the missing questions and headings;

2) Solve every problem in the space below with a graphic scheme. As the space is limited, use signs,words, and numbers. Then write the operation.

3) Why these problems have been given together? What was the most difficult? Why?

A)…… Teacher Anna Maria has marked 6 admission tests for each of her 23 pupils. How many tests has she marked?	B)..... Teacher Anna Maria has marked 138 tests. Each of her pupils has taken 6.	C)..... Teacher Anna Maria has marked 138 tests. Tests have been taken by her 23 pupils. .....



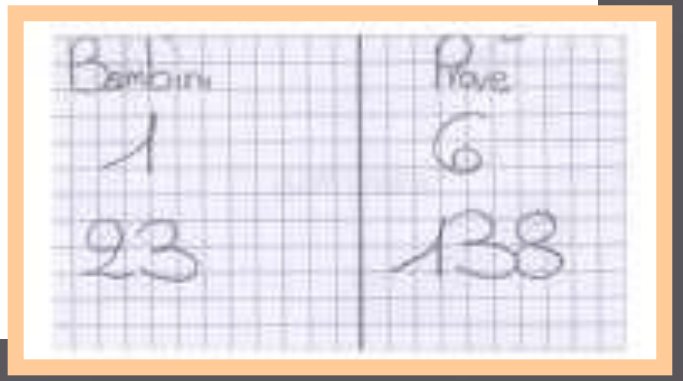
First task

Results

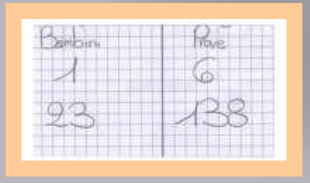
(small group work)

The missing questions of the problems are consistently stated by 7 groups among 10.

Half of the groups (5) produce solution schemes, e.g.



Children tests



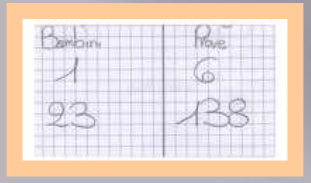
First task

Mathematical Discussion

Teacher: *Some pupils think that it is strange to start from two numbers that count tests and to get a third number that counts children. 138 count tests, 6 counts tests. How is it possible that an operation between these numbers produces a number that counts children?* [...]
Samantha: *Among those problems, it tells that each child makes 6 tests hence 1 comes out, that is [the child] who makes the tests.*
Teacher: *If I have understood well, Samantha says that it is not true that in the text B there are only numbers counting tests. She sees also a 1 that counts the child who makes 6 tests.*
Basma: *That 1 counts each child making 6 tests. [...]*
Donato: *Samantha and Alyssa are right. They make one understand that tests are 138, pupils are 23, and tests made by one pupil are 6.*
Gabriele: *Good boy! Donato is good because he has explained that 6 is the tests made by the pupils, 138 is the tests collected from all pupils and 1 is the child who makes the tests.*

B).....
Teacher Anna Maria has marked 138 tests. Each of her pupils has taken 6.

.... *How many pupils?*



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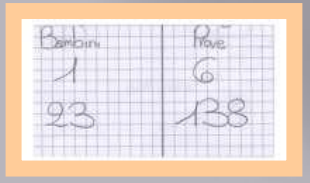


Second task

1) A child has solved the three problems building the schemes below. Cut and match each scheme to one problem, then solve it by an operation.

2) Why has the child decided to use these schemes?

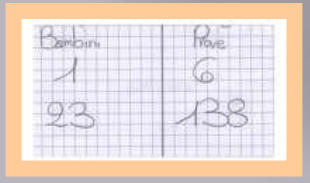
What do you think?

Mario sticks his photos in an album. He fills 9 pages and sticks 6 photos in each page. Calculate how many photos Mario has stuck.	Mario fills 9 pages of his album to stick 54 photos. Each page features the same number of photos. How many photos are stuck in each page?	Mario sticks 54 photos in his album. He sticks 6 photos in each page. How many pages has he used?

P	F
1	?
9	54

P	F
1	6
?	54

P	F
1	6
?	54



Second task

Results

(individual work)

The scheme has different functions:

It identifies:

Two “measure” spaces

The unknown

The operation

It helps to master measuring units

Semiotic mediator

Second teaching experiment (Rita Canalini)

What followed?

3. Creation of similar muliplicative problems with variation
4. Discussion about these problems (consistency)
5. Cut and paste schemes and operations to match a system of three multiplicative problems
6. Systematic recourse to schemes while solving arithmetic problems
7. Final assessment (creation)

(B. Comez, master thesis in progress)

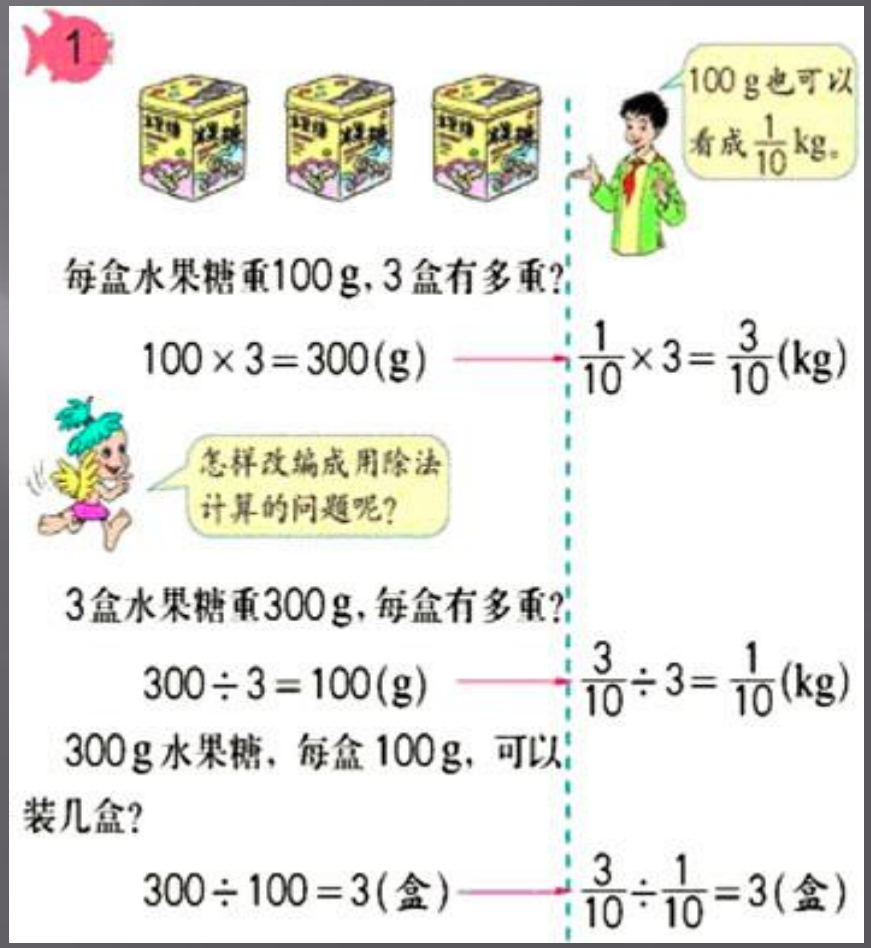
A prompt for more complex multiplicative word problems (fractions) three problems on boxes of fruit candies (6° grade)

Original problem:
Each box of fruit candies weighs 100 g.
How much do the three boxes weigh?

Variation problem 1:
Three boxes of fruit candies weigh 300 g.
How much does each box weigh?

Variation problem 2:
Fruit candies weigh a total of 300 g.
Each box contains 100 g of fruit candies.
How many boxes are needed for all the candies?

Xhuxua Sun (2011)



A comparison between the two Italian experiments

What have they in common?

Theoretical framework.

Semiotic mediation theory (Bartolini Bussi & Mariotti)

Cultural Analysis of Content (problems with variations)

Systematic exchange, between teachers, of protocols, methodologies, critical analysis (a form of “lesson study” at large), to enrich each other

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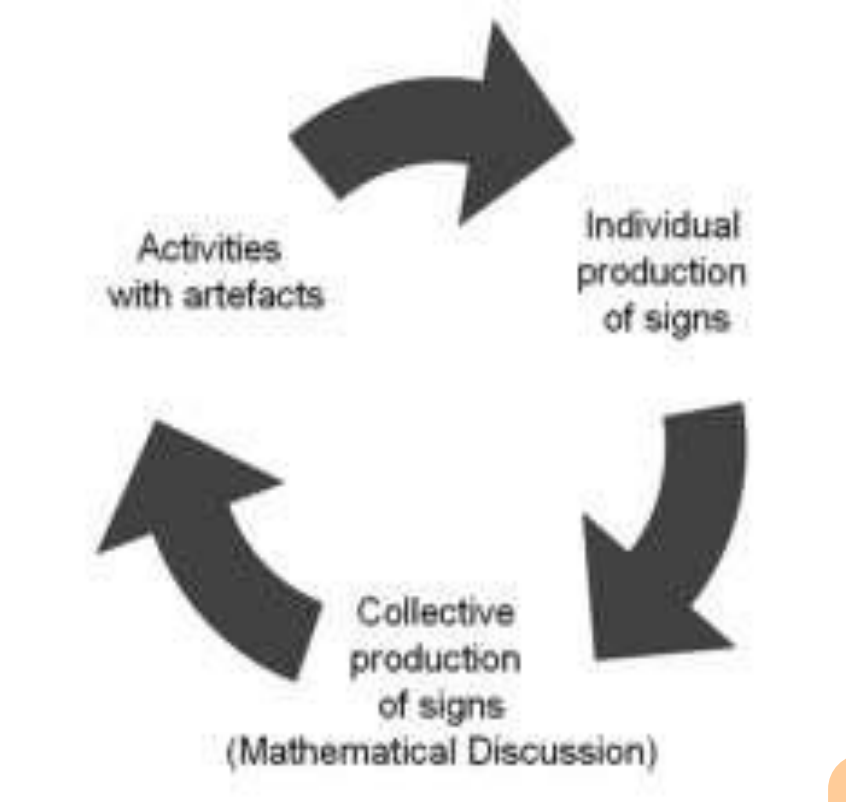
Semiotic mediation theory



Artefacts are, in this case, the very schemes, either introduced by the teacher or created by the pupils themselves, and later transformed into objects/motives of the Mathematical Discussion orchestrated by the teacher

Systematic alternation between individual – small group – whole class interaction

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Cultural Analysis of Content

Chinese problems with variations as prompt:

- the number size;
- the joint presentation of addition-subtraction and of multiplication-division;
- the great deal of problems to be solved together;
- the presence of graphic schemes;
- the high level of the request “to explain the arrangement” (the nine problems on ducks);
- the very dry and concise texts which mirror the Chinese style.

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Fragmentation or connectedness?

All the comparative studies of curricula (practices)

US – China

(western world – far east)

emphasize

Fragmentation vs. connectedness

Fragmentation is an obstacle towards algebraic reasoning

In these short Italian experiments (and in the replicas made by other teachers) we have always observed

the emergence
with awareness
of algebraic reasoning

Cultural Analysis of Content

Chinese problems with variations as prompt:

- the number size;
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- the great deal of problems to be solved together;
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- the high level of the request “to explain the arrangement” (the nine problems on ducks);
- the very dry and concise texts which mirror the Chinese style.

Dry or narrative texts?

Does a richer narrative text help or inhibit the arithmetic/algebraic solution?

Is the interpretation of a narrative text more demanding or more helpful compared to a paradigmatic text?

Does it apply in the same way to high and low achievers, to pupils of Italian extraction and to pupils coming from other cultures?

Change of beliefs?
Change of practices?
by means of Cultural Analysis of Content

Observed processes

1) to query one's own beliefs.

2) to take part in the design of innovative classroom activities where CAC is exploited.

3) to test the designed activities in one's own classrooms.

Prospective teachers in teacher education programs

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Change of practices?
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Practicing teachers in teacher education programs

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"Daring" teachers and teacher-researchers

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Prospective teachers in teacher education programs

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"Daring" teachers and teacher-researchers

A kind of tentative conclusion:

(inter)Cultural Analysis of Content
is not aimed at
copying mechanically the Chinese methodology
of problems with variation.

But rather at
querying some beliefs
of our teachers

A rock from another mountain can be used
to chisel your own jade

(Xiao Ya, Shijing: He Ming, 1000 A.C.)

Thanks

My collaborators: Rita Canalini and Franca Ferri

Patrizia Bartolini and Enrico Ferraresi (translation from Chinese textbooks)

My students (prospective primary school teachers)

The practicing teachers who took part in the workshops

The IPC of SEMT 11, which invited me

And you all for your attention!

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